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ORIGINAL RESEARCH

Tension stiffening approach in concrete of tensioned members

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Abstract This paper presents an analytical model to calibrate the tension stiffening effect of tensile reinforced concrete members. The tension stiffening behaviour is a primordial task in reinforced concrete mechanic field. In this model, the stress–strain relationship of the tension stiffening effect described in the cracking range is proposed. The application of the analytical expression for tensile reinforced concrete member aims principally to quantify the tension stiffening phenomenon in the cracking range. In this concern, a parametrical study is established, which concerns the influence on the tension stiffening behaviour of concrete strength, reinforcement ratio, bar diameter and instantaneous properties of concrete. Obtained results relative to the influence of different parameters of the analysis are shown and commented.

Keywords Tension stiffening · Analytical model · Nonlinear analysis · Tensile members · Bar diameter · Reinforcement ratio

Introduction

The intact concrete between adjacent cracks can still carry tensile stresses after cracking occurs in reinforced concrete members. This phenomenon known as the tension stiffening is principally generated due to the bond between reinforcing bars and surrounding concrete. Cracking and tension stiffening are considered among complex phenomena of reinforced concrete mechanic. In this subject, a variety of constitutive laws, many approaches and various techniques have already been proposed to predicate the tension stiffening behaviour in RC and FRP reinforced concrete structure. Many works have been shown that neglecting the tension stiffening contribution leads to soft structures. The economy in reinforcement when the tension stiffening effect is taken into account during the procedure of the design has been quantified.

Various models integrating the tension stiffening effect have been proposed to design RC structures. Among models, we can quote the model of Branson (1968), which represents the tension stiffening effect using an equivalent moment of inertia of the cracked section of the beam. This model is largely used by designers to compute RC and FRP reinforced beam deflections. However, other models are based on the modification of constitutive laws of steel or the sectional area obtaining an effective area of the section.

These models have also been used in nonlinear analysis of pull-out tests and bent beams. In this scope, we can illustrate various works published in literature: Gilbert and Warner model (1978), Choi and Cheung (1996) and the CEB manual design model (Cracking et al. 1985), and among those that modify the sectional area, such as: ACI-440 (2003) and Behfarnia (2009). Recently, many complex models based on the bond-slip mechanisms between concrete and reinforcing bars have been published: Khalfallah (2008), Gupta and Maestrini (1990), Kwak and Song (2002), Ng et al. (2011). In these models, there are some limitations in applications because these approaches depend on the nature of the distribution functions of bond along the reinforcement axis and it follows in general a series of complex integrations of the second differential equation of bond.

However, previous attempts have already been drawn to present more realistically the tension stiffening effect,

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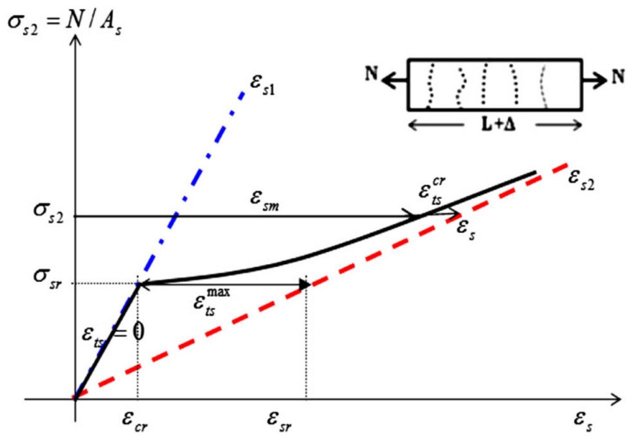

Fig. 1 Tension stiffening behaviour of tensile members

assuming the modifications taken in steel stresses. Based on this concept, this work introduces a novel tension stiffening approach applied to tensile RC members. The proposed model uses a parabolic curve to describe the post-cracking region of tensile stress–strain relationship of reinforcing bars improving the CEB-tension stiffening model (Cracking et al. 1985). In addition, the influence of concrete strength, reinforcement ratio, bar diameter and instantaneous properties of concrete on the tension stiffening behaviour is studied and commented.

CEB model (Cracking et al. 1985)

The CEB model developed for tensile RC members integrates the tension stiffening effect through an increase in reinforcement stiffness. The mechanism of cracking of RC members subjected to monotonic tensile loading is shown in Fig. 1. This figure presents different region of the structural behaviour of RC members: un-cracking concrete (I), cracking concrete (II) and yielding of reinforcement (III). The CEB model adopts a stress–strain relationship of reinforcing bars in terms of an average strain expressed by strain of un-cracked section and that of totality cracked one, respectively.

The average strain of reinforcement is expressed by:

$$\varepsilon_{sm} = \varepsilon_{s2} - \Delta\varepsilon_s, \tag{1}$$

where ε_{s2} is the strain in bare bar and $\Delta\varepsilon_s$ is the difference between totality cracked section and partially cracked reinforced concrete one (Fig. 1).

In the CEB manual design (Cracking et al. 1985), the increment of steel strain $\Delta\varepsilon_s$ based on experimental results is given as:

$$\Delta\varepsilon_s = \Delta\varepsilon_{s\max}\frac{\sigma_{sr}}{\sigma_{s2}}, \tag{2}$$

σ_{s2} and σ_{sr} are the stress in bare bar and that corresponds to the cracked section level when the maximum stress in concrete reaches the strength of concrete in tension and $\Delta\varepsilon_{s\max}$ is the maximum strain; defined as the difference between strains ε_{s1} and ε_{s2}, which occur at the beginning of the cracking process.

An efficient tension stiffening model for nonlinear analysis of reinforced concrete members has been published by Stramandinoli et al. (2008) where the relationship (1) has been developed in function of the un-cracked section ε_{s1} and the full-cracked section of the member ε_{s2}.

$$\varepsilon_{sm} = \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2 \varepsilon_{s1} + \left[1 - \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2\right]\varepsilon_{s2}. \tag{3}$$

The CEB model presents a consistent theory of the post-cracking behaviour of RC members under pure tension. The corresponding curve is characterized as a bi-linear branch adopted in the cracking of concrete region.

Proposed model

A novel expression that describes the tension stiffening behaviour for structural members is proposed in this section. The model is based on the modification of the stress–strain formula of reinforcing bars as shown below.

The concrete is assumed to behave like a linear-elastic material until its tensile strength is reached. When the applied load, N, is relatively small, the strains in steel and in concrete maintain a single value. In this phase, the strain is then given by:

$$\varepsilon = \varepsilon_s = \varepsilon_c = \frac{N}{A_s E_s + A_c E_c} \tag{4}$$

E_s and E_c are elastic modulus of reinforcing bars and concrete before cracking, respectively. A_s and A_c are the reinforcement and the concrete area, respectively, and N is the tensile applied load.

Until the formation of the primary crack of concrete, components of the composite material change its behaviour due to the initiation and propagation of cracking mechanism. At this level, the stress value of reinforcing bars is calculated based on the notion of the cracked section, where the maximum stress in the concrete under tension reaches its strength one. Equation (4) deals with:

$$\sigma_{sr} = \frac{1 + \eta\rho}{\rho} f_t \tag{5}$$

$\rho = \frac{A_s}{A_c}$ is the reinforcement ratio, $\eta = \frac{E_s}{E_c}$ and f_t is the tensile strength of concrete.

The modification added to the CEB model, founded on a bi-linear law in the cracking range, cannot represent well the structural behaviour of RC tensile members. For this reason, a novel expression improving the CEB model is proposed. Due to the dominance of the material nonlinearity, it is to be noted that the heterogeneous composition of concrete, the introduction of the tension stiffening effect, the bond phenomenon, the dowel action, etc. influence the nonlinearity of the RC member response. Based on this concept, in the post-cracking region, a polynomial expression is formulated until yielding of reinforcing bars takes place. The stress–strain relationship of the bare bar is assumed to be asymptotic straight of the curve representing the expression developed minimizing the tension stiffening in the cracking region of the member behaviour. After cracking, the stress in the reinforcement steel between the once cracking and the yielding of reinforcement is then expressed using the mathematical establishments of this concept by:

$$\sigma_s = \frac{E_s(\varepsilon_{sm} - \varepsilon_{cr})^2}{(\varepsilon_{sm} - \varepsilon_{cr}) + (\varepsilon_{sr} - \varepsilon_{cr})} + \sigma_{sr}, \tag{6}$$

where ε_{cr} and ε_{sr} are the cracking strain and the strain in the reinforcement in state II with totally cracked section without any concrete contribution corresponding to the stress σ_{sr}.

The corresponding strain can be drawn using Eq. (6) as:

$$\varepsilon_{sm} = \varepsilon_{cr} + \frac{(\varepsilon_s - \varepsilon_{sr})}{2} + \frac{1}{2}\sqrt{(\varepsilon_s - \varepsilon_{sr})(\varepsilon_s + 3\,\varepsilon_{sr} - 4\,\varepsilon_{cr})} \tag{7}$$

with:

$$\varepsilon_s = \frac{N}{E_s A_s}$$

The tension stiffening contribution in the cracking range of concrete can then be evaluated by:

$$\varepsilon_{ts}^{cr} = \frac{(\varepsilon_s + \varepsilon_{sr})}{2} - \varepsilon_{cr} - \frac{1}{2}\sqrt{(\varepsilon_s - \varepsilon_{sr})(\varepsilon_s + 3\,\varepsilon_{sr} - 4\,\varepsilon_{cr})} \tag{8}$$

Particularly, the maximum tension stiffening can be deduced as (Fig. 1):

$$\varepsilon_{ts}^{\max} = \varepsilon_{sr} - \varepsilon_{cr} \tag{9}$$

Analytical model

Validation of the model

To verify the validity of the proposed model, it's necessary to compare obtained results with those already published in literature. Experimental data elaborated by Wu and Gilbert (2009) are chosen. In this case, the applied load versus average axial strain curve is shown in Fig. 2. Wu and Gilbert have tested several reinforced concrete prisms submitted to an axial tension having a square cross section (100 mm × 100 mm) and 1,100 mm long and containing a single reinforcing bar longitudinally running through the centroid of each cross section. The tensile axial load was applied to the ends of the reinforcing bar protruding from each end of the concrete prism. Four of the specimens were tested under monotonically increasing deformation up to yielding of the reinforcing steel bar (the short-term tests) (Wu and Gilbert 2009) (Tables 1, 2).

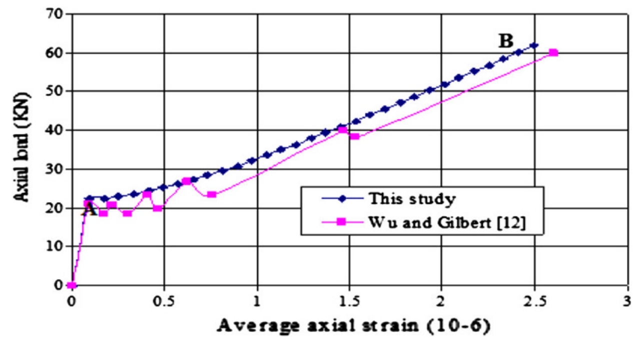

Fig. 2 Load versus average strain under monotonic loading

Firstly, based on the comparison between the analytical model and the experimental results, it seems that the obtained curve using this approach shows well concordance with the experimental one (Fig. 2).

Secondary, the cracking load obtained is 22.46 KN. It is 6.44 % higher than that obtained in Wu and Gilbert (2009). In this way, the ultimate load is evaluated as 61.81 KN; it is 3.01 % higher than the experimental data that is quantified as 21.10 kN.

Prior to the un-cracking region of the curve (Fig. 2), the specimen is at its stiffest and the load–strain curve is assumed linear. When the first cracking occurs ($P = P_{cr} = 22.46$ kN), there is an abrupt change of stiffness that continues to degrade under increasing deformation as others cracks can be occur in the cracking region (portion AB). The phenomenon of cracking primordially influences on the overall response of tensile member and on the tension stiffening behaviour. For this reason, curves show that as the load increases, the tension stiffening strain gradually reduces.

Parametric study

Influence of reinforcement ratio

It's very important to understand how the area of concrete around bars can contribute to stiff RC tensile members. Figure 3 shows a comparison of reinforcement ratios on the

Table 1 Details of the test specimens (Wu 2009)

Specimen	Concrete strength (MPa)	Bar diameter D (mm)	Dimension $b \times d \times l$ (mm)	Reinforcement ratio ρ %
C21	21	12	$100 \times 100 \times 1{,}100$	1.13
C40	40	20	$100 \times 100 \times 1{,}100$	3.14

Table 2 Material properties (Wu 2009)

Property	Reinforcing bars		Concrete strength	
	12 mm	20 mm	Grade 21	Grade 40
Strength (MPa)	500	500	21	40
Stiffness (GPa)	200	200	22.4	22.4
Tensile strength (Mpa)	–	–	2.04	3.80

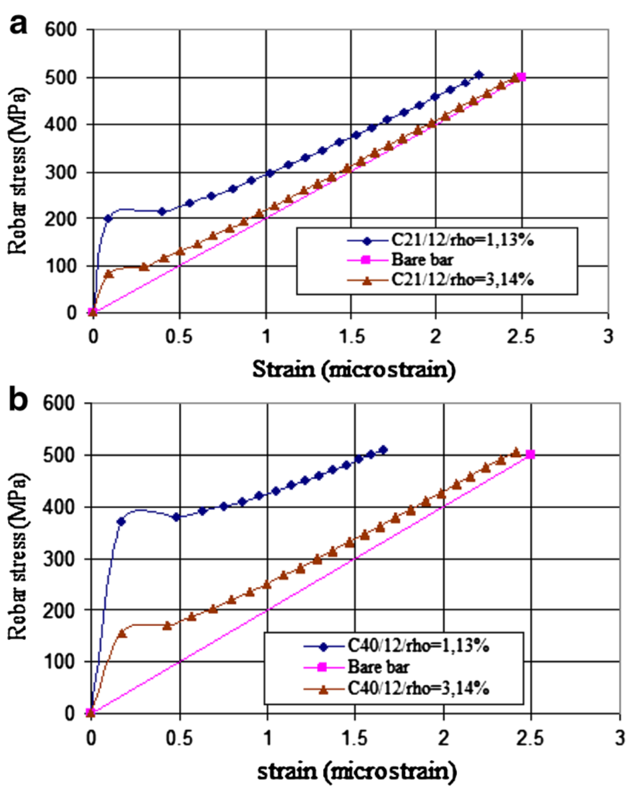

Fig. 3 Influence of reinforcement ratio on the tension stiffening. **a** $\rho = 1.13$ %, **b** $\rho = 3.14$ %

tension stiffening effect. In this study, two grades of concrete are used and obtained results are shown in separate graphs: Fig. 3a illustrates grade C21 concrete and Fig. 3b shows grade C40 concrete. The primordially remark, which can be observed, is that the tension stiffening increases with the decrease in the reinforcement ratio. In addition, the tension stiffening contribution is more pronounced with light reinforcement with high quality of concrete (Fig. 4a, b).

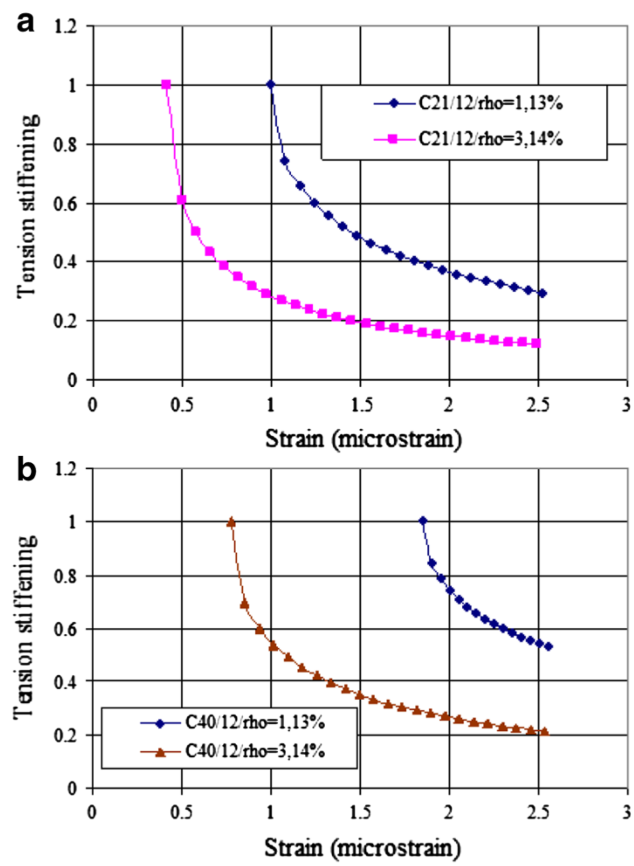

Fig. 4 Influence of reinforcement ratios. **a** C21 grade, **b** C40 grade

Influence of concrete strength

Figure 5a, b compares the influence of concrete strength on the tension stiffening effect and on the response of specimens reinforced with different reinforcement ratios. The comparison clearly shows that the effect of tension stiffening at the cracking stage decreases with increasing concrete strength.

Figure 5 compares the influence of concrete strength on the responses of RC member having 12 mm and 20 mm bar sizes. This comparison shows that the effect of tension stiffening decreased with increasing concrete strength in the stabilized cracking stage (Fig. 5a, b). Also, it is clear that high strengths of concrete reproduce great tension stiffening compared to low ones (Fig. 6).

These results affirm another time results obtained per Perera and Mutsuyoshi (2011) and Kaklauskas et al.

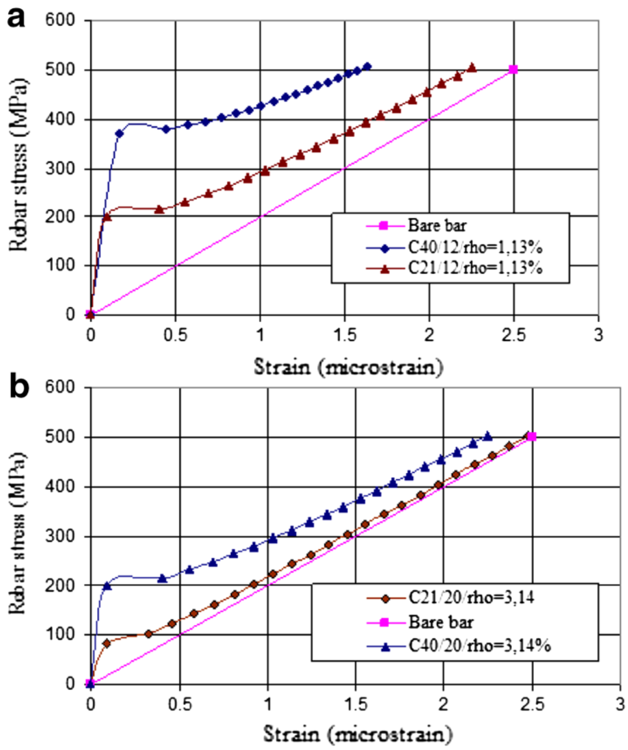

Fig. 5 Influence of concrete strength on the tension stiffening. **a** C21, **b** C40

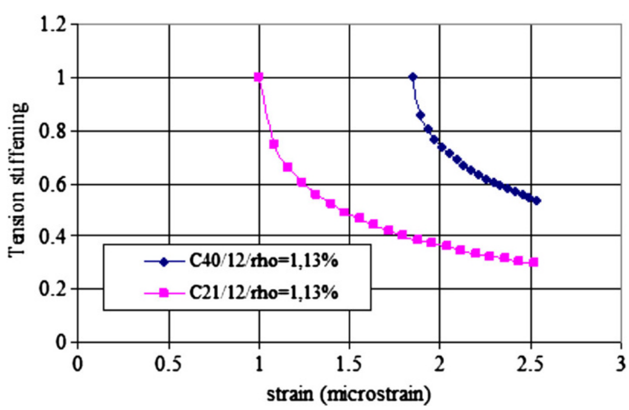

Fig. 6 Influence of concrete strength

(2011), which are opposite to Abrishami and Mitcher (1996) results; the specimens with higher strengths of concrete exhibit a larger tension stiffening in the cracking stage.

Influence of instantaneous modulus of concrete

In this case, the instantaneous modulus of concrete has a neglected effect on the tension stiffening when the tensile members are weakly reinforced (Figs. 7a, 8). But, a relative difference can be observed for members reinforced with higher reinforcement ratios.

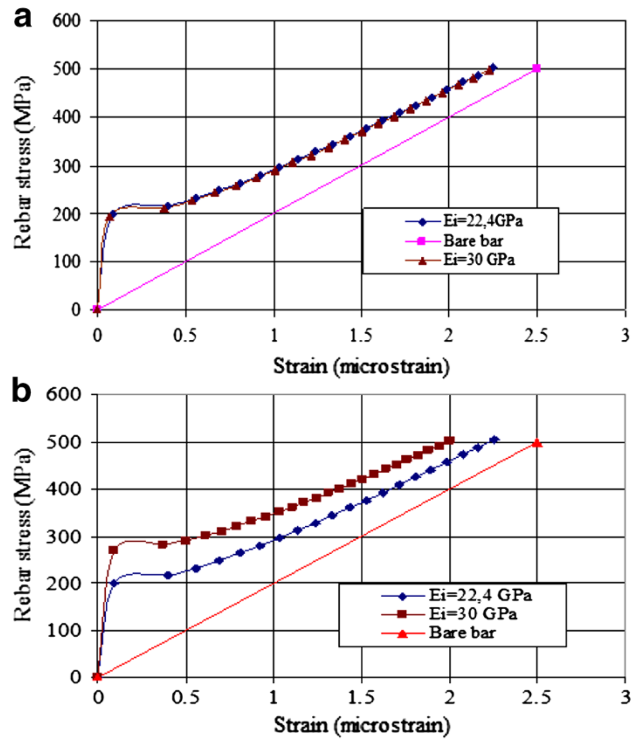

Fig. 7 Influence of instantaneous modulus of concrete. **a** $\phi = 12$ mm, **b a** $\phi = 20$ mm

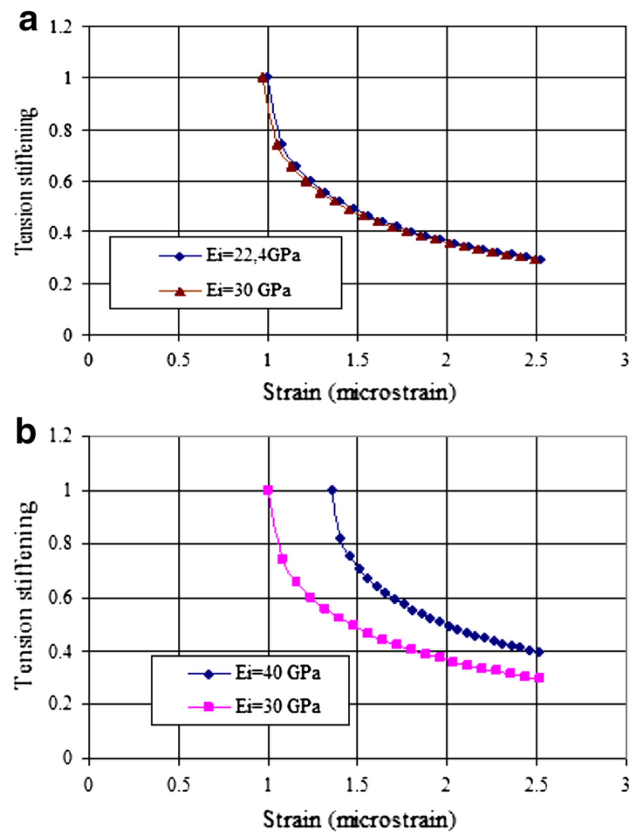

Fig. 8 Influence of instantaneous concrete modulus. **a** C21 grade, **b** 40 grade

Conclusions

In this study, an analytical expression of tension stiffening model for reinforced concrete members is presented. The relationship quantifying the tension stiffening is described using an average stress–average strain relationship in the cracking behaviour of members. Firstly, the work presents the validity of the model described in the above sections, and secondly the tension stiffening behaviour was investigated using the proposed analytical expression of the tension stiffening contribution in the cracking range.

Based on obtained results of this study, the following conclusions can be drawn:

- Concrete strength and reinforcing ratio have an influence on the tension stiffening effect. The tension stiffening was more pronounced in members with small reinforcement ratios and with high strength of concrete.
- There is no significant influence on the tension stiffening behaviour using different instantaneous modulus of concrete and different bar diameters.
- High concrete strength and bar size can provide an important ultimate load of reinforced concrete members.
- The analysis requires the introduction of concrete-reinforcement bar features to understand this aspect to accurate predictions of tension stiffening effect.

Conflict of interest The authors declare that they have no competing interests.

Authors' contributions KS has developed the model and GD has established the numericalexamples. Both, KS and GD have written together conclusions. Both authorsread and approved the final manuscript.

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